In this paper I construct a model-free implied volatility index, SVIX, from OMXS30 options based on a variance replication technique, independent of any option pricing model. The SVIX index exhibits several stylized properties of volatility indices such as long memory components, mean reversion and volatility clustering. The relationship between OMXS30 returns and SVIX is negative, with some indication of an asymmetric component. There is some evidence that implied volatility, represented among other by SVIX, is superior to historical volatility in predicting future volatility and there is a contemporaneous volatility transmission between VIX and SVIX. In addition, I construct another index, SSVIX, based on simple variance swap replication which can be hedged and priced even if we allow for jumps in the underlying asset.

A thesis submitted in fulfillment of the requirements for the degree of Master of Science in Finance at University of Gothenburg, School of Business, Economics Law

December 2015
"If you hear a "prominent" economist using the word 'equilibrium,' or 'normal distribution,' do not argue with him; just ignore him, or try to put a rat down his shirt."

- Nassim Nicholas Taleb, The Black Swan: The Impact of the Highly Improbable
Acknowledgements

I am grateful for the input and goodwill I have received from other people, without your help it wouldn’t been possible to write this paper. First, I would like to thank Harry Matilainen at the SIX-Group for helping me with the option data needed for this thesis. Secondly, I would like to thank my sister Linn and my friend Daniel for valuable discussions on coding. Last but not least, I would like to thank my supervisor, Dr. Adam Farago, for all valuable comments, input and guidance along the way . . .
# Contents

## Acknowledgements

## Contents

<table>
<thead>
<tr>
<th>Section</th>
<th>Pages</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 Introduction</td>
<td>1</td>
</tr>
<tr>
<td>2 Previous research</td>
<td>3</td>
</tr>
<tr>
<td>2.1 Research on volatility and volatility indices</td>
<td>3</td>
</tr>
<tr>
<td>2.1.1 Stylized properties of volatility</td>
<td>3</td>
</tr>
<tr>
<td>2.1.2 Implied volatility and IV indices as a forecast of future volatility</td>
<td>4</td>
</tr>
<tr>
<td>2.1.3 Relationship between returns and volatility indices</td>
<td>5</td>
</tr>
<tr>
<td>2.1.4 Volatility spillover effects in Equity markets</td>
<td>6</td>
</tr>
<tr>
<td>3 Theory and construction of SVIX</td>
<td>7</td>
</tr>
<tr>
<td>3.1 Theory behind SVIX</td>
<td>7</td>
</tr>
<tr>
<td>3.1.1 Introduction to variance swaps</td>
<td>8</td>
</tr>
<tr>
<td>3.1.2 Valuing variance swaps using replication technique</td>
<td>9</td>
</tr>
<tr>
<td>3.1.3 Limitations and critique of variance swap replication</td>
<td>11</td>
</tr>
<tr>
<td>3.1.4 Valuing simple variance swaps using replication technique</td>
<td>11</td>
</tr>
<tr>
<td>3.2 Practical construction of SVIX and SSVIX</td>
<td>12</td>
</tr>
<tr>
<td>3.2.1 Step 1. Selection of options to include</td>
<td>12</td>
</tr>
<tr>
<td>3.2.2 Step 2a. Calculation of variance for SVIX</td>
<td>13</td>
</tr>
<tr>
<td>3.2.3 Step 2b. Calculation of variance for SSVIX</td>
<td>14</td>
</tr>
<tr>
<td>3.2.4 Step 3. Inter- or extrapolation of variance</td>
<td>14</td>
</tr>
<tr>
<td>3.2.5 Final calculation of SVIX and SSVIX</td>
<td>15</td>
</tr>
<tr>
<td>4 Data</td>
<td>16</td>
</tr>
<tr>
<td>4.1 Input data for SVIX calculations</td>
<td>16</td>
</tr>
<tr>
<td>4.1.1 Input price data: Bid-Ask or Close prices?</td>
<td>16</td>
</tr>
<tr>
<td>4.1.2 Data restrictions and missing observations</td>
<td>17</td>
</tr>
<tr>
<td>5 Results</td>
<td>18</td>
</tr>
<tr>
<td>5.1 The SVIX series</td>
<td>18</td>
</tr>
<tr>
<td>5.1.1 Sub period 1 Jan-2005 - May 2006 - Low volatility regime</td>
<td>18</td>
</tr>
<tr>
<td>5.1.2 Sub period 2 May 2006 - Aug 2008 - Mid volatility regime</td>
<td>19</td>
</tr>
<tr>
<td>5.1.3 Sub period 3 Aug 2008 - Jul 2009 - High volatility regime</td>
<td>19</td>
</tr>
<tr>
<td>5.1.4 Sub period 4 Jul 2009 - Jul 2011 - Mid volatility regime</td>
<td>20</td>
</tr>
</tbody>
</table>
### 5.1 Sub periods

<table>
<thead>
<tr>
<th>Sub period</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>5 Jul 2011 - Jun 2012 - High volatility regime</td>
<td>20</td>
</tr>
<tr>
<td>6 Jun 2012 - Jun 2015 - Low volatility regime</td>
<td>20</td>
</tr>
</tbody>
</table>

### 5.2 Descriptive statistics

- Page 21

### 5.3 Tests of statistical properties

- Page 21

### 5.4 Forecast quality of the SVIX index

- Page 24

### 5.5 Relationship between OMXS30 returns and SVIX

- Page 27

### 5.6 Spillover effects in volatility indices

- Page 29

### 5.7 The relationship between SVIX and SSVIX

- Page 30

### 6 Conclusion

<table>
<thead>
<tr>
<th>Concluding remarks</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>32</td>
</tr>
</tbody>
</table>

### A Appendix

<table>
<thead>
<tr>
<th>Normality tests</th>
<th>34</th>
</tr>
</thead>
<tbody>
<tr>
<td>Autocorrelation</td>
<td>35</td>
</tr>
<tr>
<td>Spillover effects in volatility indices - plot of VIX, VSTOXX and SVIX</td>
<td>35</td>
</tr>
<tr>
<td>Comparison of SVIX and SIXVX</td>
<td>36</td>
</tr>
<tr>
<td>SVIX - OMXS30 relationship</td>
<td>37</td>
</tr>
<tr>
<td>Option data used in the SVIX/SSVIX calculation</td>
<td>38</td>
</tr>
</tbody>
</table>

### Bibliography

- Page 39
Chapter 1

Introduction

A central concept within asset pricing is the uncertainty of asset returns. The most common way to determine the uncertainty of an asset is to estimate its volatility. Within a financial setting, volatility is most often defined as the standard deviation of asset returns:

\[ \hat{\sigma} = \frac{1}{T-1} \sqrt{\sum_{t=1}^{T} (R_t - \bar{R})^2} \]

Although volatility cannot be translated directly into risk, it can serve as a risk measure. Since \( \hat{\sigma} \) (the sample standard deviation) is distribution free, Poon and Granger (2003) argue that it is useless to use \( \sigma \) as a risk measure unless it is connected to a distribution or pricing dynamic. Most often, investors assume a normal distribution for returns when \( \sigma \) is used as a risk measure (Poon and Granger (2003)). The concept of risk and return is very central within asset pricing and portfolio theory. This has led to comprehensive research on the subject, with many ways to estimate uncertainty, or volatility, of financial assets. Henceforth, this uncertainty will be referred to as volatility.

Many methods have been proposed to properly estimate volatility, including stochastic and deterministic time-series methods such as Realized and Implied volatility models. Implied volatility (IV) is derived from an option price and thus shows what the market "implies" about future volatility of the underlying asset. In the Black-Scholes\textsuperscript{1} model, volatility is one of six inputs, but the only which is not observable in the market itself (Black and Scholes (1973)). Thus, the Black-Scholes implied volatility is what the volatility of the underlying asset "must-be", given all the other observable variables. Models based on Black-Scholes options pricing formula have been used to extract the volatility.

\textsuperscript{1}I discuss more on the BS assumptions in A.4 Appendix together with a description of the BS model dependent IV index SIXVX which I use in this paper.
Chapter 1. Introduction

implied volatility from market prices, while in later years, model-free implied volatility (MFIV) have grown in popularity. One drawback of Black-Scholes implied volatility is that it depends on all underlying assumptions of the Black-Scholes model. What distinguishes Model-free IV from Black-Scholes IV, is that it relies on a replication technique of a variance swap and does not rely on the same strict assumptions as the Black-Scholes model. MFIV indices have been constructed on a historical as well as live-updated basis, providing investors with expected volatility in a coming period, derived from market option prices. The most known index of this kind is represented by the VIX, provided by CBOE since 1993. VIX measures the 30-day expected market volatility implied by S&P 500 index options and relies on a variance replication technique to capture the price of variance. There has been a lot of research on properties of MFIV indices, how well they predict future realized volatility and how they compare to other volatility models for forecasting. Several stylized facts about volatility have been established, e.g. mean-reversion, long-memory components, stationarity and non-normality.

In this paper, I construct a MFIV index, SVIX, representing the expected 30-day volatility implied by Swedish index option prices. The index is constructed on a daily basis from January 2005 to June 2015 using observations on OMXS30 index options. The theory and methodology is based on the same variance replication technique that is used for the VIX. After constructing the SVIX, I investigate its statistical properties, how it relates to the Swedish equity market, its information content on future realized volatility and how these findings can be interpreted. In addition, I construct a second index, SSVIX, which relies on a replication technique of simple variance swaps (inspired by Martin (2013)) and has been proposed to more properly capture the variation in index movements and to allow for jumps in the underlying asset’s price.

Similar papers have been conducted on the Swedish market by Reuterhall (2005) and Dahlman and Wallmark (2007), but on earlier time periods. With this paper, I aim to first provide an updated version of a MFIV index on the Swedish stock market ranging from 2005-2015, including the latest financial crisis. I provide a broad overview of many different aspects of this index (e.g. statistical properties, relationship with underlying index and predictive power). Secondly, there still does not (to my knowledge) exist any official MFIV index on the Swedish stock market and as mentioned by Dahlman and Wallmark (2007), research on smaller equity market volatility is scarce. Thirdly, there has not been any research covering an MFIV index based on simple variance swap replication (my SSVIX index), nor any constructed index of such kind on the Swedish market.
Chapter 2

Previous research

2.1 Research on volatility and volatility indices

This part will outline previous academic research on the subject, relevant to volatility and volatility indices. The first part will cover research on general statistical properties of volatility and volatility indices. The second part will cover implied volatility and how well it predicts future volatility. The third part covers the relationship between volatility indices and index returns. The last part covers volatility transmission in international equity markets.

2.1.1 Stylized properties of volatility

There are several well documented characteristics of volatility of financial time series, including non-normality, excess kurtosis, volatility clustering, mean reversion and autocorrelation. Andersen and Bondarenko (2007) find that the sample autocorrelation pattern is very slowly decaying, supporting the hypothesis that volatility processes contain long memory components. The same result is found by Ahoniemi (2006) on the logarithmic level series of the VIX together with negative autocorrelation in the log VIX first differenced series which point towards mean reversion. Early papers by Perry (1982) and Pagan and Schwert (1990) conclude that volatility time series have a unit root, while newer papers (e.g. Ahoniemi (2006), Dahlman and Wallmark (2007) Skiadopoulos (2004)) conclude volatility series to be stationary. Granger et al. (2000) find volatility of intra-day returns to experience a long memory with autocorrelations that are significantly above zero, for up to a thousand lags or more. In a study of the Greek derivative market, Skiadopoulos (2004) finds evidence for autocorrelation and mean reversion for the Greek implied Volatility Index (GVIX).
Chapter 2. Previous research

2.1.2 Implied volatility and IV indices as a forecast of future volatility

An implied volatility index is constituted of observations on implied volatility, extracted from historical and current option prices. Thus, real-time implied volatility can be compared to historical implied volatility. CBOE VIX is probably the most well known index representing 30 day market expected volatility. But can implied volatility indices really be considered forward looking or should they be taken as a mere indicator of fear? If volatility indices are good estimates of future realized volatility, they should provide unbiased estimates of future realized volatility. Whaley (2000) concludes, over a fourteen year period, that VIX/VXO has acted reliably as a fear gauge and that high levels of the VIX has been followed by market turmoil, however it should be noted that Whaley’s analysis is made on the old VIX/VXO construction. Simon (2003) writes that if implied volatility indices rather represents ‘investor fear gauge’ they will incorporate investors emotions. Thus, after a large market drop such indices would reflect investors increased demand to buy put options. He finds evidence that the VXN index (representing the implied 30-day volatility of Nasdaq 100 options) averages around 7-1/2 percentage points higher than subsequent realized volatility. Shaikh and Padhi (2015) performs a similar analysis on the India VIX and reach the conclusion that the India VIX represents both the ‘investor fear gauge’ as well as being the best unbiased estimate of future stock market volatility. González and Novales (2009) estimates a daily volatility index for the Spanish market using the Eurex method, used to estimate the German and Swiss volatility indices VDAX-NEW and VSMI. They reach the conclusion that volatility indices capture investors current attitude towards risk, but are not very useful to predict future behavior of realized volatility over longer periods. Similar results are found by Skiadopoulos (2004) who constructs an implied volatility index, GVIX, for the Greek derivative market. He reaches the conclusion that GVIX cannot forecast future FTSE/ASE-20 returns and cannot be treated as a leading indicator for the stock market. However, Skiadopoulos (2004) finds results in line with Whaley (2000), Giot (2002) and Simon (2003), that the index can be seen as ‘investor fear gauge’.

Implied volatility extracted from option prices will give the market expectation of future volatility. Christensen and Prabhala (1998) concludes that “If option markets are efficient, implied volatility should be an efficient forecast of future volatility, i.e., implied volatility should subsume the information contained in all other variables in the market information set in explaining future volatility”. Numerous studies have been conducted on whether implied volatility is a good predictor of future volatility with rather mixed conclusions. Poon and Granger (2003) find that option implied volatility contains most information about future volatility and is superior to historical, GARCH and stochastic volatility models. By using a data sample of S&P 100 index options from November 1983
to May 1995, Christensen and Prabhala (1998) also find implied volatility to be superior to historical volatility in forecast quality. They also reach the conclusion that implied volatility contains incremental information beyond that of historical volatility. Results derived by Jiang and Tian (2005) on S&P 500 index options suggests that model-free implied volatility is a more efficient predictor for future realized volatility and incorporate all information contained in the BS implied volatility and past realized volatility. Carr and Wu (2006) find that VIX has predictive power in future realized variance and that GARCH models do not provide additional information when VIX is included as a regressor in the model.

On the other hand, contradicting results are found by Canina and Figlewski (1993), Lamoureux and Lastrapes (1993) and Day and Lewis (1992). Canina and Figlewski (1993) find that implied volatility is a poor forecast of realized volatility and that implied volatility does not incorporate information contained in recent observed volatility. Day and Lewis (1992) find mixed results when comparing forecast quality of implied volatility with conditional volatility from GARCH and EGARCH models and state that neither of the models completely characterize conditional stock market volatility.

2.1.3 Relationship between returns and volatility indices

Already in 1976, Fischer Black documented a relationship between stock returns and volatility changes. Black (1976) determines that a drop in firm value will cause returns of its stock to be negative and most often increase the leverage of the firm. If the debt-equity ratio rises (assuming fixed debt), Black concludes that an effect of this will be a rise in volatility of the stock. This effect has been referred to by academics as "the leverage effect". Figlewski and Wang (2000) study this effect further and find evidence that the suggested "leverage effect" is more of a "down market effect" that may have not as much connection to firm leverage as believed. They find a strong "leverage effect" connected to falling stock prices but also many deviations that question the leverage changes as the explanation. Further they state that those effects are much smaller or even nonexistent when positive stock returns reduce leverage. This suggests that the relationship is asymmetric, something pointed out by Whaley (2000). As mentioned above, he argues that VIX can be used as a barometer of investors fear of downside risk, and as investors greed in markets trending upwards. If there should be an asymmetric relationship, negative returns should cause larger changes in implied volatility indices than equal sized positive returns. Other empirical work that support this asymmetric relationship is conducted by Bollerslev and Zhou (2006), Fleming et al. (1995), Simon (2003) and Giot (2005) but results of no asymmetry is found by Giot (2002) and Dahlman and Wallmark (2007). The negative relationship between volatility indices and returns
has been widely documented. However, results on whether there exists an asymmetric component are not as clear cut and seem to differ between markets and time periods.

2.1.4 Volatility spillover effects in Equity markets

There are several studies that find results of volatility transmission between different equity markets. Factors that have lead to increasing spillover effects between markets have often been that markets have become more integrated and that trade flows are multidirectional. Baele (2005) states that "Increased trade integration, equity market development, and low inflation are shown to have contributed to the increase in EU shock spillover intensity". He finds evidence for volatility transmission from the US market to a number of European markets during high world volatility regimes. The same result, that volatility spillover increases in high volatility regimes, have been concluded by King and Wadhwani (1990) as well. Results derived by Ciffarelli and Paladino (2012) state that the Latin American and European markets are highly sensitive to news in the US markets, while Hong Kong was the only market that experienced volatility spillover from the US market. From Granger causality tests, they conclude that there is no clear cut hierarchy in causality of volatility transmission among the worlds equity markets, even if US markets are often placed on top of the causal pyramid.

Badshah (2009) finds significant spillover effects between VIX, VXN, VDAX and VS-TOXX with bi-directional causality which covers the implied volatility transmission. He finds evidence that markets within the same region are of high integration, but even markest in different regions show signs of integration in implied volatility. However, Aboura (2003) finds evidence that the French and German implied volatility indices show less correlation with each other than with the US index which is in contrast to the previous studies above. Skiadopoulos (2004) concludes that there is a contemporaneous spillover of changes in implied volatility between VIX and GVIX but no lead effects.

\footnote{For the interested reader I refer to Ciffarelli and Paladino (2012) page 52 to read more about results on ranking of causality.}
Chapter 3

Theory and construction of SVIX

3.1 Theory behind SVIX

The theory behind the construction of a MFIV index, is based on Demeterfi et al. (1999) who covers valuation of variance swaps and how a variance swap can be replicated by a hedged portfolio of options with suitable strikes. The CBOE VIX index relies on the same theory with some practical adaptions. First I will briefly cover the theory behind variance swaps and then move on to how one can use a variance replicating portfolio to value these instruments. The theoretical discussion will end with some limitations which arise when relying on the framework of Demeterfi et al. (1999) and how those may affect this paper. After outlining the theoretical part, I will move on to explain the method used to construct the SVIX.

Just as investors trade stocks for the exposure of stock prices movements, investors may trade volatility or variance because they believe to have some insight of future volatility. However, investors who want a pure exposure to volatility cannot make use of single options on an index or stock due to the fact that such options provide exposure to more variables than the volatility. By examining the option greeks in Black-Scholes (Hull (2000)), we find that options provide exposure to stock price movements (Delta), volatility, (Vega), time (Theta), interest rates (Rho) as well as several higher order partial derivatives of the Black-Scholes PDE which can be computed. Demeterfi et al. (1999) concludes that delta-hedging is at best inaccurate due to the violation of many of the Black-Scholes assumptions; volatility cannot be accurately estimated, stocks cannot be traded on a continuously basis, liquidity problems and so on.

A MFIV index aims to capture market expected implied variance, or generally return variation over the coming 30 day period under a so called risk-neutral measure. However,
volatility is stochastic, which makes a MFIV index to typically differ from an expected return variation under an actual or objective measure. Thus, a MFIV index will not provide a pure forecast of volatility for the underlying asset, but rather bundles a forecast with market pricing of the uncertainty surrounding the forecast. An effect of this, is that implied volatilities will include premiums, compensating investors for systematic risk due to equity-index volatility (Andersen and Bondarenko (2007)).

### 3.1.1 Introduction to variance swaps

To provide a pure exposure to volatility, investors can make use of volatility swaps which are forward contracts on future realized volatility, or variance swaps which are similar forward contracts on the square of future volatility. Demeterfi et al. (1999) further states that investors mainly talk about volatility, "but it is variance that has more fundamental theoretical significance”. This is due to the fact that the correct way to value a swap contract, is to value it’s replicating portfolio. Variance swaps can more reliably be replicated, by using portfolios of options at varying strike prices. The variance rate between time 0 and time \( T \) can be replicated using a portfolio of put and call options (Hull (2000)). The replication technique of variance swaps will make ground for the construction of SVIX where, due to non-arbitrage conditions, the fair value of a variance swap is determined by the cost of the replicating portfolio of options (Demeterfi et al. (1999)). The SVIX index will then be calculated as the square root of the price of variance.

A swap contract on variance provides investors with a clean exposure to variance, and as with all swap contracts, the initial risk-neutral value is zero. Variance swap contracts are forward contracts on annualized variance with a payoff at expiration equal to \( \sigma^2_R - K_{var} \times N \) where \( \sigma^2_R \) is realized variance (expressed in Equation (3.1)) of the underlying asset over the contract life, \( K_{var} \) is the delivery price for variance and \( N \) is a notional amount in dollars per annualized variance point. Thus, the payoff to the holder of the contract receive \( N \) times the points of which the realized variance sigma has differed from the delivery price \( K_{var} \).

So the variance swap will be a contract to exchange:\(^1\)

\[
\sigma^2_R = \left( \log \frac{S_1}{S_0} \right)^2 + \left( \log \frac{S_2}{S_1} \right)^2 + ... + \left( \log \frac{S_T}{S_{T-1}} \right)^2 \tag{3.1}
\]

\(^1\)The realized variance during a time period can be expressed more formally as the sum in Equation (5.1).
for some fixed "strike" $\tilde{V}$ at time $T$, where the market convention is to set $\tilde{V}$ so that no money is exchanged by the initiation of the contract. The expectation of Equation 3.1 in the $\Delta \rightarrow 0$ limit converges to

$$\tilde{V} = \mathbb{E}^* \left[ \int_0^T (d \log S_t)^2 \right]$$

(3.2)

Following the derivation in [Martin (2013)], the strike on a variance swap is determined by the price of a notional contract paying the log of the asset’s simple return at time $T$.

$$\tilde{V} = 2rT - 2 \mathbb{E}^* \log \frac{S_T}{S_0}$$

(3.3)

[Demeterfi et al. (1999)] derives the equation to price this contract in terms of European call and put options as

$$e^{-rT} \mathbb{E}^* \log \frac{S_T}{S_0} = rTe^{-rT} - \int_{0}^{F_{0,T}} \frac{1}{K^2} put_{0,T}(K) dK - \int_{F_{0,T}}^{\infty} \frac{1}{K^2} call_{0,T}(K) dK$$

(3.4)

By substituting (3.4) back into (3.3) we will then end up with the general result under assumptions 1-5 as

$$\tilde{V} = 2e^{rT} \left\{ \int_{0}^{F_{0,T}} \frac{1}{K^2} put_{0,T}(K) dK + \int_{F_{0,T}}^{\infty} \frac{1}{K^2} call_{0,T}(K) dK \right\}$$

(3.5)

3.1.2 Valuing variance swaps using replication technique

Consider a portfolio of index options. This portfolio will provide exposure to both Delta, as the index level moves, as well as Vega, measuring the variance of the index return. To achieve a portfolio which depends only on variance, the Delta must first be neutralized which can be done using an index forward contract with matching time to maturity. Once the portfolio has been Delta-hedged, its value will solely be dependent on variance movements (under Black-Scholes assumptions, Demeterfi et al. (1999) shows that the effect of a negative Theta is offset by the Gamma). The sensitivity to variance of such a portfolio will not be constant as I show in my example in Figure 3.1, the Vega is highest

---

3 Assumption 1-5: 1. European puts and calls can be traded at arbitrary strikes. 2. The underlying asset does not pay dividends. 3. The continuously compounded interest rate is constant. 4. The underlying asset and the risk-free bond can be traded continuously in time. 5. The underlying asset price follow an Itô process $dS_t = rS_t dt + \sigma S_t dZ_t$ under the risk-neutral measure.
for at-the-money options and drops dramatically as the options go in-the-money or out-of-the-money. To achieve a constant sensitivity to variance movements, a wide range of options with different strike prices must be weighted inversely proportional to the square of the strike price $K^2$. An intuitive explanation to this is that as the index level moves to higher values, each additional option of higher strike in the portfolio will provide an additional contribution to the variance sensitivity proportional to that strike. An option with higher strike will therefore produce a variance sensitivity contribution that increases with the index level. In addition, the contributions of all options overlap at any definite index level. Therefore, to offset this accumulation of index level dependence, one needs diminishing amounts of higher-strike options, with weights inversely proportional to $K^2$ \(\text{Demeterfi et al.} \ (1999)\). The idealized version, in continuous time, of the SVIX index looks the following from the derivation above

$$SVIX^2 = \frac{2e^{rT}}{T} \left\{ \int_{0}^{F_{0,T}} \frac{1}{K^2} \text{put}_{0,T}(K) \, dK + \int_{F_{0,T}}^\infty \frac{1}{K^2} \text{call}_{0,T}(K) \, dK \right\} \quad (3.6)$$

\[\text{Figure 3.1: Vega exposure - Left figures show call option Vega as a function of stock price. Right figures show the sum of variance exposure for a portfolio of the same options, weighted inversely proportional to the square of their individual strike price}\]
3.1.3 Limitations and critique of variance swap replication

To perfectly replicate a variance swap, it takes an infinite number of correctly weighted strikes, which in practice is impossible. There is only a finite and discrete number of options even in the most liquid market, which imposes some limitations to applying the replication technique of variance swaps in theory. As seen in Figure 3.1, when more strikes are added and the spaces between each strike are fewer, the Vega straightens out and becomes almost constant. However, in the tails of available strikes and below/above those, the replication will be imperfect and make the portfolio of options less responsive than a true variance swap. Another issue is the fact that indices, and to a higher extent, stocks undergo jumps, which is discussed by Carr and Wu (2006) and Martin (2013). The VIX index squared has a very concrete economic interpretation and can be seen as either the price of a portfolio of options, or as an approximation of a variance swap rate up to the discretization error discussed above and the error induced by jumps (Carr and Wu (2006)). Martin (2013) states that in presence of jumps, the relationship between VIX and the quadratic variation breaks down and hence, VIX$^2$ does not correspond to a fair strike of a variance swap and the replicating portfolio does not replicate a variance swap payoff. To investigate the importance of this effect, I also calculate the SSVIX index following the derivation found in Martin (2013). The SSVIX is introduced in the next section.

3.1.4 Valuing simple variance swaps using replication technique

Variance swaps cannot be hedged when we allow for jumps in the series and market participants have had to impose caps on payoffs. Martin (2013) writes "These caps which have become since 2008, the market convention in index variance swaps as well as single name variance swaps, limit the maximum possible payoff on a variance swap, but further complicate the pricing and interpretation of the contract. A fundamental problem with the definition of a conventional variance swap can be seen very easily: if the underlying asset, and individual stock goes bankrupt, so that $S_t$ hits zero at some point before expiry $T$, then the payoff is infinite.". In this section I will shortly outline how to make use of Simple Variance Swaps and use a similar replication technique as discussed in the previous section to value a Simple Variance Swap in the presence of jumps. Following Martin (2013), a Simple Variance Swap is an agreement to exchange

$$\left( \frac{S_1 - S_0}{F_{0,0}} \right)^2 + \left( \frac{S_2 - S_1}{F_{0,1}} \right)^2 + \ldots + \left( \frac{S_T - S_{T-1}}{F_{0,T-1}} \right)^2$$

(3.7)
for a pre-determined strike $V$ at time $T$, where $F_{0,t}$ is the forward price of the underlying index or stock at time $t$, known at time 0.

The continuous time pricing and hedging of a simple variance swap in the $\Delta \to 0$ limit simplifies to

$$SSVIX^2 = \frac{2e^{rT}}{T} \left\{ \int_0^{F_{0,T}} \frac{1}{F_{0,T}^2} put_{0,T}(K) dK + \int_{F_{0,T}}^\infty \frac{1}{F_{0,T}^2} call_{0,T}(K) dK \right\}$$

(3.8)

For those interested in the complete derivation and theory of Simple Variance Swap pricing and hedging, please see Martin (2013).

The difference between the continuous SVIX and continuous SSVIX definition is that the weights in SSVIX contains $F_{0,T}^2$ instead of $K^2$. This can easily be seen by comparing Equations (3.6) and (3.8)

### 3.2 Practical construction of SVIX and SSVIX

We saw the continuous models of SVIX and SSVIX in the previous section. In this section I will outline how one can use the discretized versions to achieve the desired results. I will describe both the SVIX and SSVIX methods, which are very similar up to the point of the variance calculation which will be clear in the following sections.

#### 3.2.1 Step 1. Selection of options to include

Options included in the calculation of the SVIX are out-of-the-money OMXS30 calls and puts centered around an at-the-money strike price $K_0$. For each day SVIX is to be calculated, this reference strike needs to be determined first. Since options with exactly 30 days to maturity are usually not available, a 30-day variance will be inter- or extrapolated from options with shorter/longer maturities. The options are divided into "near-term" and "next-term" expiring in the two nearest months. CBOE uses options with more than 23 days and less than 37 days to expiration, including both standard 3rd Friday expiration options as well as weekly options. In 2014, CBOE started to include weekly options, something I cannot do in this paper due to data limitations.

The first step is to determine a forward OMXS30 level, which is done by identifying the smallest absolute difference between calls and puts at each specific strike price. Prices are

---

Please note that what I refer to as SVIX in this paper, is the "normal" implied volatility index based on variance swap replication and should not be confused with what Martin (2013) refers to as SVIX. In his paper, his SVIX is based on the Simple Variance Swap replication, which in this paper is referred to as SSVIX.
mid-quoted bid/ask prices. Using the defined strike where the put/call price difference is smallest for near-term and next-term options, the forward level $F$ is calculated as

$$F = \text{Strike price} + e^{RT} \times (\text{call price} - \text{put price}) \quad (3.9)$$

where:

- **Strike price** is the identified option strike which minimizes the $|\text{call price} - \text{put price}|$
- $R$ is the continuously compounded risk free interest rate
- $T$ is time to maturity in days
- **call price** is the call option price identified at **Strike price**
- **put price** is the put option price identified at **Strike price**

The forward OMXS30 level, can be seen as an expected at-the-money expiry level of the options. Given the reference point, $F$, $K_0$ can be determined - the strike price immediately below the forward index level $F$ for both near- and next-term options. We define the strike price of the $i^{th}$ option as $K_i$ and the price corresponding to that strike as $Q(K_i)$. Using the criteria below, we chose put options for strike prices below $K_0$, and call options for strike prices above $K_0$. At $K_0$, both call and put prices are chosen and their prices are averaged. That is,

$$Q(K_i) = \begin{cases} 
\text{put price} & \text{for } K_i < K_0 \\
(\text{put price} + \text{call price})/2 & \text{for } K_i = K_0 \\
\text{call price} & \text{for } K_i > K_0 
\end{cases}$$

### 3.2.2 Step 2a. Calculation of variance for SVIX

When the correct option strips are chosen according to the $Q(K_i)$, the variance for near- and next term options can be calculated where $\sigma_1^2$ is calculated using near-term and $\sigma_2^2$ calculated using next-term options. The below equations are discretized versions of (3.6) where $\sigma_1^2$ and $\sigma_2^2$ are calculated using near-term and next-term options respectively.

$$\sigma_1^2 = \frac{2}{T_1} \sum_{i=1}^{h} \frac{\Delta K_i}{K_i^2} e^{RT} Q(K_i) - \frac{1}{T_1} \left[ \frac{F_1}{K_0} - 1 \right]^2 \quad (3.10)$$

$$\sigma_2^2 = \frac{2}{T_2} \sum_{i=1}^{h} \frac{\Delta K_i}{K_i^2} e^{RT} Q(K_i) - \frac{1}{T_2} \left[ \frac{F_2}{K_0} - 1 \right]^2 \quad (3.11)$$
where $T$ is time to expiration for the $i_{th}$ option, $K^2$ is the squared strike of the $i_{th}$ option and $\Delta K$ is the difference between the strike of $K_i$ and $K_{i-1}$ defined as follows

$$
\Delta K_i \begin{cases} 
K_i - K_{i-1} & \text{for } K_i = K_{max} \\
(K_{i+1} - K_{i-1})/2 & \text{for } K_{max} > K_i > K_{min} \\
K_{i+1} - K_i & \text{for } K_i = K_{min}
\end{cases}
$$

Hence, a single option’s contribution to the variance is proportional to $\Delta K$ and to the price of the option, and inversely proportional to the square of that option’s strike price. The last term to the right in the equation adjusts for the difference between $F$ and the nearest available strike.

### 3.2.3 Step 2b. Calculation of variance for SSVIX

The discretized version of the Simple Variance Swap replication for SSVIX will be very similar to the one used to calculate SVIX. The only difference is the denominator in the sum part, which is exchanged for $F_i$. Since $F_i$ is a constant for each given $T$ the variance calculation simplifies to

$$
\sigma_1^2 = \frac{2}{T_1} \sum_{i=1} \frac{\Delta K_i}{F_{T_0}^2} e^{RT} Q(K_i) - \frac{1}{T_1} \left[ \frac{F_1}{K_0} - 1 \right]^2
$$

$$
\sigma_2^2 = \frac{2}{T_2} \sum_{i=1} \frac{\Delta K_i}{F_{T_0}^2} e^{RT} Q(K_i) - \frac{1}{T_2} \left[ \frac{F_2}{K_0} - 1 \right]^2
$$

These equations are discretized versions of (3.8) and apart from the variance construction, SSVIX follows the same steps as SVIX. The same rule is applied here, $\sigma_1^2$ and $\sigma_2^2$ are calculated using near-term and next-term options respectively.

### 3.2.4 Step 3. Inter- or extrapolation of variance

To make sure that the SVIX index reflects a 30-day market expected variance, the variance calculations needs to be inter- or extrapolated for options with shorter or longer maturities

$$
\sigma^2 = \left[ T_1 \sigma_1^2 w + T_2 \sigma_2^2 (1 - w) \right] \frac{365}{30}
$$
where $T_1$ and $T_2$ are time to maturity in years for the two contract months $\sigma^2_1$ and $\sigma^2_2$ are variances annualized for the periods corresponding to $T_1$ and $T_2$ and $w$ is a weight assigned to the first option strip to get a 30-day variance

$$w = \frac{T_2 - 30}{365}$$

For those days where both the near-term and next-term options are maturities larger than 30 days, the weights will be larger than one or less than zero, since the inter- or extrapolation is of linear shape.

### 3.2.5 Final calculation of SVIX and SSVIX

After the inter- or extrapolation, SVIX and SSVIX can finally be calculated as

$$SVIX = 100\sqrt{\sigma^2}$$

$$SSVIX = 100\sqrt{\sigma^2}$$
Chapter 4

Data

4.1 Input data for SVIX calculations

In this part I will present the data needed for the SVIX calculations and modifications needed to be done to ensure the reliability and quality of the data.

To construct an IV index, one need to obtain option prices for a wide range of strike prices. I obtained data on OMXS30 option prices from SIX\(^1\) which includes daily observations on a large range of strike prices for put and calls. Prices are divided into bid, ask and close, where information exists, and obviously there are far more bid/ask than close prices. The moneyness of the options serve as an indicator of how liquid they are. E.g. deep in-the-money options, are far less liquid and therefore have fewer close prices observed. The sample period ranges from January 2005 to June 2015. The options used are of European style, i.e. they can only be exercised at maturity. Table A.3 in Appendix shows the amount of options used to calculate SVIX and SSVIX.

A proxy for the risk-free rate is used and matched with each observation date. The proxy used for risk-free rate is the STIBOR-1M obtained from Sveriges Riksbank and is expressed as continuously compounded\(^2\).

4.1.1 Input price data: Bid-Ask or Close prices?

When constructing the index, one has to chose whether to use an average of bid-ask or the actual closing prices. There are pros and cons using both methods, but for this

---

1SIX is a leading provider of financial information within Europe and specializes in processing and distributing financial data for institutional investors http://www.six.se/en/about-us.

2In theory, a risk-free rate with exact match of the days to maturity should be used, but due to practical limitations I have made use of the STIBOR-1M as an approximation.
paper I have chosen to work with an average of bid-ask, and will explain the rationale for this below.

For the VIX construction, CBOE uses midquoted bid-ask prices. Previous studies by Fleming et al. (1995) conclude that to avoid negative autocorrelation implied in close prices caused by the bid-ask bounce, it is better to use midquoted bid-ask prices. Further, Lamoureux and Lastrapes (1993) make use of the midpoint of bid-ask prices and states that "actual transaction prices may include price pressure effects, and (latest) transaction prices from both markets at a fixed point in time (such as closing) will always be asynchronous". On the other hand Skiadopoulos (2004) uses close prices as he finds much stronger negative autocorrelation in the case when GVIX is constructed by the average of bid-ask prices and suggests that close prices should be superior.

I find the synchronicity argument to be of most importance mainly due to the fact that OMXS30 options are not that liquid. Consider a day with large index movements, observing an option with moneyness far from unity. It may be the case that there has been only one settlement in the morning and after that the index has moved significantly. The close price will then poorly represent the index movements and settlement level, while an average of bid-ask prices at the end of the day will better capture this change and settlement level. Another, more practical reason, for using midquoted bid-ask prices is the fact that my dataset contains far more bid-ask prices than close prices. If I were to make use of close prices, there would be a significant amount of missing days in the SVIX index.

4.1.2 Data restrictions and missing observations

There are days where neither SVIX nor SSVIX can be constructed due to missing observations in the dataset. This may be due to that no matching put and calls are found to identify $F_i$, or that $F_i$ has been identified, but there are only one observed put or call at either side of $F_i$. There needs to be at least two observed option prices on either side of $F_i$ (or $K_0$) in order to come up with a $\Delta K$, hence if there is only one observed option price, no difference can be calculated. For VIX, CBOE uses options with at least eight days of maturity, since options closer to maturity might be affected by expiry effects and thus introduce noise to the index. However, since my dataset does not include that many options, I set the restriction at two days of maturity to not lose to many observations.

---

3I chose to drop days with less than two differences in either side. For the call options this is 20 days, and for the puts only seven days, i.e. a very small part of the entire sample. More on this can be found in section A.6 Appendix.
Chapter 5

Results

5.1 The SVIX series

In the first part of this Chapter I will go through the SVIX series together with the underlying index, OMXS30, for the time period and point out events which have had a significant impact on the SVIX level. Figure 5.1 shows the OMXS30 on the left-hand axis vs. SVIX on right-hand for the period 2005-01-03 to 2015-06-11. The time period has been divided into six sub periods which are characterized by low, medium or high volatility environments and also selected in combination with significant periods characterizing bull/bear markets for the OMXS30. The overall SVIX level mean is 22.89 with a sample standard deviation of 9.46. I will not cover the SSVIX index in this first part since its movements in relation to the OMXS30 are very similar to those of SVIX. The topic of interest regarding the SSVIX, is how it relates to SVIX since they are computed in two different ways. This will be covered later in the result section.

5.1.1 Sub period 1 Jan-2005 - May 2006 - Low volatility regime

The first period is characterized by a stable SVIX level with a low sub period mean of 15.07. Observing the OMXS30 we can see a steady increase in this period of 36%. By the end of this period the OMXS30 drops from the peak in April 2006 at 1076 to levels slightly below 900. Looking at the SVIX, we find a dramatic increase in the SVIX relating to this period with a first peak at 42.75. The sub period standard deviation of SVIX (the volatility of volatility) is the lowest for the entire sample at 1.80.
5.1.2 Sub period 2 May 2006 - Aug 2008 - Mid volatility regime

Apart from the first month, where the OMXS30 drops some 100 points and SVIX reaches its first peak, this period was quite stable. Looking at the OMXS30 we see a steady increase up to the highs before the financial crisis of 2008-09 and relatively stable volatility. Although we find some spikes in the SVIX together with increased variance of the SVIX by the end of 2007, the mean SVIX level during this period remained at 24.34, i.e. slightly above average. SVIX sub period standard deviation remained 5.56.

5.1.3 Sub period 3 Aug 2008 - Jul 2009 - High volatility regime

Bear markets following the financial crisis of 2008 pushed the SVIX index to the highest levels in my sample, which not surprisingly coincide with the bottom of the crisis. The first peak, found at September 15th 2008 coincides with Lehman Brothers filing for Chapter 11 bankruptcy protection. After the Lehman crash, implied volatilities reached extreme levels which made the SVIX reach a new peak of 81.70. During this period the SVIX mean was 43.14, i.e. 88% higher than the overall mean. SVIX sub period standard deviation was 12.88, i.e. this period was characterized by both volatile asset prices as well as high volatility of volatility. We will see later that the differences between SVIX and SSVIX during this time period is by far the largest ones.
5.1.4 Sub period 4 Jul 2009 - Jul 2011 - Mid volatility regime

During this sub period, the OMXS30 increased by 43%. The SVIX sub period mean was 23.01, i.e. at the overall mean level. There are some smaller spikes in the SVIX level in mid-2010 but both the mean and variance of SVIX remained relatively low overall. This was a period of medium volatility in line with the average SVIX level, but less volatile SVIX level compared to the entire sample.

5.1.5 Sub period 5 Jul 2011 - Jun 2012 - High volatility regime

In the wake of the financial crisis of 2008, many European Union countries faced severe problems to refinance their government debt or to bail-out national banks. Market turmoil over great sovereign debts and possible insolvency of the financial system, together with poor economic outlooks, made markets take a down-turn by the end of summer 2011. We can also see that the SVIX peaks at 60.12 and then continue to stay at quite high levels, around 40, for some time before a drop in the beginning of 2012. During this period, the negative relationship between the OMXS30 and SVIX becomes evidently clear with the last peaks of the SVIX around April 2012. The SVIX volatility was 7.46 which is slightly below the sample volatility.

5.1.6 Sub period 6 Jun 2012 - Jun 2015 - Low volatility regime

The last period of the sample contains a relatively stable period for the SVIX level and upward-trending bull-market for the OMX30, reaching all-time-high levels in April 2015. During this period, the SVIX mean was 17.02 while the OMXS30 increased by 59%. By observing this period, we get a hint of the asymmetrical relationship between implied volatility indices and their underlying indices. While OMXS30 grew steadily, the SVIX remained fairly constant with a lower mean compared to the entire period. The asymmetrical relationship will be examined more rigorously later in this chapter. The volatility of SVIX during this period was low, at 3.24.
Chapter 5. Results

5.2 Descriptive statistics

Table 5.1 shows the overall summary statistics for all indices and each respective differenced series. VSTOXX is an implied volatility index calculated using the same method as VIX, but with EURO STOXX 50 as underlying. Comparing SVIX with SSVIX we see that SSVIX has a slightly lower mean but are still very close to each other. SVIX has also had a higher mean than VIX but lower than VSTOXX during the sample period. Finally we compare SVIX with SIXVX and find that SVIX has higher mean and standard deviation for the levelled series. For the differenced series, SVIX has more than double standard deviation compared to SIXVX. The reason for higher standard deviation in my series (SVIX and SSVIX) might be due to some noise introduced by weak restrictions on e.g. maturity but also due to fewer data points.

Table 5.1: Summary statistics

<table>
<thead>
<tr>
<th>index</th>
<th>level series</th>
<th>differenced series</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>N</td>
<td>mean</td>
</tr>
<tr>
<td>SVIX</td>
<td>2,505</td>
<td>22.89</td>
</tr>
<tr>
<td>SSVIX</td>
<td>2,505</td>
<td>22.48</td>
</tr>
<tr>
<td>VIX</td>
<td>2,608</td>
<td>19.81</td>
</tr>
<tr>
<td>VSTOXX</td>
<td>2,655</td>
<td>23.52</td>
</tr>
<tr>
<td>SIXVX</td>
<td>2,500</td>
<td>20.80</td>
</tr>
</tbody>
</table>

5.3 Tests of statistical properties

This part will present some basic statistical tests commonly performed when working with time series. First I will evaluate whether the level and differenced series follow a normal distribution, are stationary and if they exhibit autocorrelation, volatility clustering and mean-reversion.

Figure A.1 in Appendix A shows histograms for the SVIX and SSVIX level and differenced series. The distributions of the level series are right-skewed, while the distributions for the differenced series are leptokurtic. To ensure the statistical significance of the non-normality I perform a Jarque-Bera test under the null-hypothesis that the series are normally distributed. The alternative hypothesis is that the data is non-normally distributed.

---

1 EURO STOXX 50 is a stock index designed by STOXX, an index provider owned by Deutsche Borse Group and SIX Group. I obtain SIXVX data from SIX Group. Data on VIX and VSTOXX is obtained from Bloomberg.

2 A more thorough discussion about the relationship between SVIX and SIXVX is found in Appendix A.4.
distributed. The null is rejected at the 1% significance level for all series. These results are in line with previous studies on the well documented non-normality of volatility.

Table 5.2: Normality tests

<table>
<thead>
<tr>
<th></th>
<th>Jarque-Bera</th>
<th>Skewness</th>
<th>Kurtosis</th>
</tr>
</thead>
<tbody>
<tr>
<td>SVIX</td>
<td>5,151***</td>
<td>1.98</td>
<td>8.80</td>
</tr>
<tr>
<td>SSVIX</td>
<td>3,703***</td>
<td>1.82</td>
<td>7.71</td>
</tr>
<tr>
<td>ΔSVIX</td>
<td>9,183***</td>
<td>0.68</td>
<td>32.64</td>
</tr>
<tr>
<td>ΔSSVIX</td>
<td>4,341***</td>
<td>0.07</td>
<td>23.40</td>
</tr>
</tbody>
</table>

Numbers in parentheses denote t-statistics based on Newey-West HAC robust standard errors. *, **, *** denotes significance at 10 percent, 5 percent and 1 percent respectively.

Stationarity has also been a widely documented property of volatility and is a necessity for mean reversion. To test whether SVIX, SSVIX and their differenced series contain a unit root I run Dickey-Fuller tests. The null-hypothesis in the Dickey fuller is that the series contain a unit root, whereas the alternative hypothesis is that the series does not contain a unit root. Table 5.3 shows the outcome of each respective test; the null of a unit root is rejected in each case.

Table 5.3: Unit root tests

<table>
<thead>
<tr>
<th></th>
<th>test stat(obs)</th>
<th>N</th>
</tr>
</thead>
<tbody>
<tr>
<td>SVIX</td>
<td>ADF (lag = 1)</td>
<td>-10.11***</td>
</tr>
<tr>
<td>SSVIX</td>
<td>ADF (lag = 1)</td>
<td>-9.71***</td>
</tr>
<tr>
<td>ΔSVIX</td>
<td>ADF (lag = 1)</td>
<td>-72.13***</td>
</tr>
<tr>
<td>ΔSSVIX</td>
<td>ADF (lag = 1)</td>
<td>-70.20***</td>
</tr>
</tbody>
</table>

Numbers in parentheses denote t-statistics based on Newey-West HAC robust standard errors. *, **, *** denotes significance at 10 percent, 5 percent and 1 percent respectively.

Figure 5.2 shows the autocorrelation structure of the level SVIX series and the first differenced SVIX series. As seen from the sample autocorrelation of the level SVIX series, the autocorrelations decay extremely slow which is in line with the well documented property of volatility being long a memory process. The first differenced series exhibit negative autocorrelation which may be supporting evidence of mean-reversion. It might be important to point out that mean reversion is a stylized fact in asset price volatility and not in asset returns. Commonly, the result of negative autocorrelation in asset returns is taken as evidence of mean-reversion, although it must not be the case.

I will not dig deeper into this topic since previous research has established that volatility

\[3\] I re-ran the adftest using a larger number of lags. The null hypothesis is rejected for lags up to 28, which all are significant at a 5% level.

\[4\] For the interested reader I refer to Choe et al. (2007) and Hillebrand (2003) for further discussion on mean reversion of asset returns. Choe et al. (2007) state that the necessity of negative autocorrelation in one-period returns is an artifact of assuming an AR(1) process for the transitory components of the underlying asset. They show that the first-order autocorrelation sign can be positive under a different lag specification of the transitory components of asset prices.
processes are mean reverting. My results of negative first-order autocorrelation in the differenced volatility series points towards the same result. These results are in line with results found by e.g. Andersen and Bondarenko (2007) and Ahoniemi (2006).

Table 5.4 shows the first three autocorrelation coefficients for each respective series together with the Ljung-Box Q statistic. The Ljung-Box test for general autocorrelation in the residuals under the null-hypothesis that the residual series do not exhibit autocorrelation. The results clearly state that the null that all autocorrelation coefficients up to the specified lag are simultaneously zero is rejected, i.e. the series exhibit autocorrelation up to the specified number of lags. In the differenced series there is significant negative autocorrelation for the first lag at 1% level and for the second lag at 5% level, which is in line with the previously mentioned mean reversion property of volatility. The Ljung-Box test statistics also show that we can reject the null of all autocorrelations coefficients up to 7, 25 or 50 lags simultaneously being zero. The evidence of high autocorrelation in the SVIX/SSVIX series indicates that the SVIX series exhibit volatility clustering.

Table 5.4: Autocorrelation test

<table>
<thead>
<tr>
<th></th>
<th>coefficients lag 1-3</th>
<th>Ljung-Box Q stat</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\rho(1)$ $\rho(2)$ $\rho(3)$</td>
<td>lags (7) lags (25) lags (50)</td>
</tr>
<tr>
<td>SVIX</td>
<td>0.922*** 0.899*** 0.886***</td>
<td>13,339*** 40,160*** 69,303***</td>
</tr>
<tr>
<td>SSVIX</td>
<td>0.928*** 0.903*** 0.887***</td>
<td>13,478*** 40,122*** 69,553***</td>
</tr>
<tr>
<td>$\Delta$SVIX</td>
<td>-0.351*** -0.067** 0.035</td>
<td>355*** 385*** 477***</td>
</tr>
<tr>
<td>$\Delta$SSVIX</td>
<td>-0.327*** -0.058** -0.009</td>
<td>297*** 327*** 395***</td>
</tr>
</tbody>
</table>

Numbers in parentheses denote t-statistics based on Newey-West HAC robust standard errors. *, **, *** denotes significance at 10 percent, 5 percent and 1 percent respectively.
With the results from tests in this section, we can establish that the SVIX series is stationary, exhibits non-normality, volatility clustering and mean reversion.

5.4 Forecast quality of the SVIX index

Numerous studies have been conducted on whether implied volatility is a good predictor of future realized volatility. The results have been pointing in different directions depending on time and data for the study. I compute the annualized 30 day realized volatility according to

$$RV_{t,t+30} = 100 \times \sqrt{\frac{365}{30} \sum_{i=1}^{30} \left[ \log \left( \frac{S_{t+i}}{S_{t+i-1}} \right) \right]^2}$$

(5.1)

To evaluate the information content and quality of SVIX in predicting future realized volatility, I follow the procedure of Christensen and Prabhala (1998) and Christensen and Hansen (2002). I run the following regressions to asess the information content in implied volatility of future volatility on a non-overlapping sample with 84 observations.

$$RV_{t+1,t+30} = \alpha_0 + \beta_1 SVIX_t + e_t$$

(5.2)

$$RV_{t+1,t+30} = \alpha_0 + \beta_2 SSVIX_t + e_t$$

(5.3)

$$RV_{t+1,t+30} = \alpha_0 + \beta_3 RV_{t-29,t} + e_t$$

(5.4)

$$RV_{t+1,t+30} = \alpha_0 + \beta_4 SIXVX_t + e_t$$

(5.5)

$$RV_{t+1,t+30} = \alpha_0 + \beta_1 SVIX_t + \beta_3 RV_{t-29,t} + e_t$$

(5.6)

$$RV_{t+1,t+30} = \alpha_0 + \beta_2 SSVIX_t + \beta_3 RV_{t-29,t} + e_t$$

(5.7)

$$RV_{t+1,t+30} = \alpha_0 + \beta_3 SIXVX_t + \beta_3 RV_{t-29,t} + e_t$$

(5.8)

where $RV_{t+1,t+30}$ is the realized future volatility at time $t$, $SVIX_t$ is the implied 30 day expected volatility at the beginning of period $t$, $SSVIX_t$ is the implied 30 day expected volatility derived from simple variance swap replication at time $t$ and $RV_{t-29,t}$ is the historical realized volatility at time $t$. Christensen and Prabhala (1998) states that there are three hypotheses that can be tested from these regressions. The first is to asess if implied volatility contain any information about future realized volatility, $\beta_1$ should be

\[ multiculturality \]
significantly different from zero. Second, if implied volatility should provide *unbiased* forecasts of future realized volatility, the intercept $\alpha_0$ should be zero and the coefficient for implied volatility $\beta_1$ should equal unity. Third, if implied volatility provides *efficient* forecasts, the residuals $e_t$ should be white noise and not correlated with any variable in the information set. Figure 5.3 shows a plot with volatility on y-axis of my different volatility series with the 84 observations.

![HRV, FRV, SVIX, SSVIX & SIXVX plot](image)

**Figure 5.3**: Historical RV in red, Future RV in blue, SVIX in dotted black, SSVIX in dotted purple and SIXVX in dotted cyan

Table 5.5 shows the separate regression results from the equations above. As mentioned before, some earlier studies concluded that implied volatility is a poor forecast of future volatility. However, some of these studies have been criticized since they make use of overlapping data. I run the forecasting regressions and divide my sample into 84 observations. From the DW-statistic we see that there is no autocorrelation among the residuals as the DW statistics are not significant at any level. From Equation (5.2) we can see that the coefficient for SVIX$_t$ is 0.789 and significant at a 1% level, with an adjusted $R^2$ of 0.542. This result implies that SVIX contains information about future volatility, however it appears to be a biased estimate. By looking at the F-value, we can reject the joint hypothesis of $\alpha_0 = 0$ and $\beta_1 = 1$. From results on Equation (5.3) we can see that the coefficient for SSVIX$_t$ is 0.862 and significant at 1% level with an adjusted $R^2$ of 0.538, thus SSVIX also contains information about future volatility. SSVIX compared to SVIX seems to provide a *slightly* less biased forecast since the coefficient is closer to unity, however the adjusted $R^2$ drops. Results from Equation (5.4) shows that historical volatility also contains information on future volatility with a coefficient of 0.697 and significant at a 1% level. However, compared to SVIX and

---

6 I end up with 84 observations where I have data for all 84 days for all variables.
SSVIX, historical volatility provide a more biased forecast with a lower adjusted $R^2$. Results from Equation (5.5) show that SIXVX seem to provide the least biased estimate with a coefficient of 0.948 and significance at a 1% level and highest $R^2$ of 0.635. Based on the results from Equations (5.2) - (5.5) it looks like that the implied volatility indices are superior to historical volatility in predictive power of future volatility.

When SVIX is included as an additional regressor to historical volatility, the coefficient for historical volatility drops to 0.279 and is only significant at a 10% level with an adjusted $R^2$ of 0.571. Similar results are found from Equation (5.7), when SSVIX is included as an additional regressor. The coefficient for historical volatility drops to 0.283 and shows only significance at a 10% level, with an $R^2$ of 0.566. When SIXVX is included as a regressor additional to historical volatility, the coefficient for historical volatility is not significant at a 10% level. However, the SIXVX coefficient is still close to unity and significant at a 1% level with a $R^2$ of 0.644. Also these results point towards that the implied volatility indices are superior to historical volatility since the significance of $RV_{t−29,t}$ drops to 10% and the coefficient drops by more than 50%.

Table 5.5: Regression results - Volatility forecasting using non-overlapping data

<table>
<thead>
<tr>
<th>Eq.</th>
<th>intercept</th>
<th>SVIX$_t$</th>
<th>SSVIX$_t$</th>
<th>RV$_{t−29,t}$</th>
<th>SIXVX$_t$</th>
<th>adj $R^2$</th>
<th>DW</th>
<th>F-value</th>
<th>N</th>
</tr>
</thead>
<tbody>
<tr>
<td>(5.2)</td>
<td>5.701*** (2.246)</td>
<td>0.789*** (0.101)</td>
<td>0.542</td>
<td>1.73</td>
<td>94.5</td>
<td>84</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(5.3)</td>
<td>4.397** (2.429)</td>
<td>0.862*** (0.114)</td>
<td>0.538</td>
<td>1.77</td>
<td>57.6</td>
<td>84</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(5.4)</td>
<td>7.454*** (2.443)</td>
<td>0.697*** (0.113)</td>
<td>0.497</td>
<td>1.98</td>
<td>79.1</td>
<td>84</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(5.5)</td>
<td>4.330*** (1.414)</td>
<td>0.948*** (0.064)</td>
<td>0.635</td>
<td>1.77</td>
<td>218.9</td>
<td>84</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(5.6)</td>
<td>5.037** (2.564)</td>
<td>0.531*** (0.125)</td>
<td>0.279* (0.161)</td>
<td>0.571</td>
<td>1.90</td>
<td>50.9</td>
<td>84</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(5.7)</td>
<td>4.234* (2.597)</td>
<td>0.574*** (0.145)</td>
<td>0.283* (0.159)</td>
<td>0.566</td>
<td>1.92</td>
<td>54.2</td>
<td>84</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(5.8)</td>
<td>4.307*** (1.602)</td>
<td>0.012 (0.169)</td>
<td>0.935*** (0.1481)</td>
<td>0.644</td>
<td>1.92</td>
<td>351.7</td>
<td>84</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Numbers in parentheses denote t-statistics based on Newey-West HAC robust standard errors.
*, **, *** denotes significance at 10 percent, 5 percent and 1 percent respectively.

In summary, my results show evidence of implied volatility, represented by SVIX, SSVIX and SIXVX, being superior to historical volatility in predicting future volatility. All of the explanatory variables provide efficient forecasts. However, all of them provide biased forecasts with adjusted $R^2$ between 50-60%. These results are in line with Dahlman and Wallmark (2007) who also finds SVIX to be superior to historical volatility with biased forecast and $R^2$ around 50%. Having established that IV contain information on future volatility, I will move on to investigate the relationship in the next section.
5.5 Relationship between OMXS30 returns and SVIX

The negative relationship between IV indices and their underlying indices has been established by many previous studies. Figure 5.1 gives a hint of that the same result holds for SVIX and OMXS30. The correlation between $\Delta$SVIX and OMXS30 (cc) daily returns expressed as $OMXS30ret_t = 100 \times (\ln OMXS30_t - \ln OMXS30_{t-1})$, for the entire sample is $-0.32$ (p-vale 0.000) and ranges around the same value for all sub periods as seen in Table A.1. These results show slightly less correlation than results presented by Dahlman and Wallmark (2007) which find negative correlation between SVIX and OMXS30 of -0.43 while Carr and Wu (2006) finds negative correlation of -0.78 between VIX and S&P500 returns. On the other hand, Skiadopoulos (2004) finds as low correlation as -0.17 between the Greek IV index and FTSE/ASE-20 returns. Dahlman and Wallmark (2007) states that lower correlation might be an effect of lower liquidity in the option markets. It is reasonable to assume that less liquid markets will yield fewer option observations to match the index, and thus lead to lower correlation.

Following Skiadopoulos (2004) I also perform a Granger causality test to establish if OMXS30 returns are caused by changes in SVIX or the other way around. It may be reasonable to assume that changes in the OMXS30 level causes option prices to change, and hence SVIX, although causality is highly philosophical. The null under the Granger causality test states that $y$ does not granger cause $x$, and if the F-value is larger than the critical value we can reject the null. As seen in Table 5.6 the results are unidirectional, i.e. that changes in OMXS30 return cause changes in the SVIX level and not vice-versa.

$$\begin{align*}
\text{Table 5.6: Granger causality tests} \\
\text{Null hypothesis} & \quad \text{F-value} & \text{Crit-val} & \text{lags} \\
\Delta SVIX \text{ does not Granger cause OMXS30ret} & 2.81 & 3.85 & 10 \\
OMXS30ret \text{ does not Granger cause } \Delta SVIX & 34.89^{***} & 2.38 & 10 \\
\end{align*}$$

Numbers in parentheses denote t-statistics based on Newey-West HAC robust standard errors. *, **, *** denotes significance at 10 percent, 5 percent and 1 percent respectively.

Whether there exists an asymmetric effect of volatility indices on asset returns has been widely discussed with different conclusions. As discussed in the previous research section, there might be an asymmetric component in implied volatility indices which Figure 5.1 might show evidence of. During times when the SVIX level remain fairly constant, OMXS30 seems to yield positive returns, while when changes in SVIX are positive, the OMXS30 seems to drop and yield negative returns. If losses in the OMXS30 lead to an increase in the SVIX which are larger than equally sized positive returns, there might be evidence for an asymmetric relationship. To assess whether there exists an asymmetric
relationship I follow the methodology of Whaley (2000). I run the following regression to test this relationship

$$\Delta SVIX_t = \alpha_0 + \beta_1 OMXret_t + \beta_2 OMXret_t^- + \epsilon_t$$  (5.9) 

Where $OMXret_t^-$ is equal to $OMXret_t$ if $OMXret_t < 0$ and zero otherwise.

If there should exist an asymmetric relationship, the coefficient for $OMXret_t^-$ should be positive and significantly different from zero. The coefficient for $OMXret_t$ expresses the standard relationship between daily OMXS30 returns and changes in SVIX level.

As seen in Table 5.7, the coefficient for $OMXret_t$ is $-0.768$ and significant at 1% level. I.e. if OMXS30 yield positive (cc) daily return of 1% unit, SVIX decreases on average 0.768 ppts. The coefficient for $OMXret_t^-$ is negative and not statistically significant at any level. Table A.2 in Appendix shows complete results for every sub period. I run the same test in each sub period to see if the results are different in low/high volatility regimes. The $OMXret_t$ coefficient is negative significant in most cases at 5% level, while $OMXret_t^-$ is only positive and significant for sub period 6.

<table>
<thead>
<tr>
<th>Table 5.7: Regression results - asymmetric relationship</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
</tr>
<tr>
<td>------------</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td></td>
</tr>
</tbody>
</table>

Numbers in parentheses denote t-statistics based on Newey-West HAC robust standard errors. *, **, *** denotes significance at 10 percent, 5 percent and 1 percent respectively.

The results from the entire period clearly point out the negative relationship between index returns and changes in SVIX. These results are in line with many other studies on the negative relationship between index returns and their volatility indices, e.g. Giot (2002) and Whaley (2000). However, the coefficient for $OMXret_t^-$ is not significantly different from zero for the entire sample period, suggesting that there should be no significant asymmetric effect between the two. However, sub period six, which is a low volatility regime, show results of an asymmetric component since the $OMXret_t^-$ is positive and significant at a 1% level. This means that for days when OMXS30 yield negative returns, the combined effect on SVIX from both independent variables is 2.034, while when OMXS30 yield positive returns the effect on SVIX level is only 1.139.

Results derived by Dahlman and Wallmark (2007) implies that there exists an asymmetric component during low volatility regimes. An explanation for this might be that demand for put options as insurance rises in low volatility regimes, but not when volatility already is high. Giot (2005) states that the rationale behind this is that traders are unwilling to bid more aggressive prices during high volatility regimes even if markets
are falling. This reasoning could be applied to results derived in sub period 6, however, looking at the entire period there is not enough evidence to conclude that there exists an asymmetric component in the SVIX index considering the negative and insignificant OMXret\_t coefficient. Similar results was concluded by Giot (2005) for the VXN index and by Dahlman and Wallmark (2007) who do not find enough evidence for an asymmetric relationship when looking at the entire sample period.

### 5.6 Spillover effects in volatility indices

In this section I examine the possible volatility transmission between the US and the European markets to the Swedish one. There is no obvious causality between which market causes volatility in other markets, however, it is reasonable to assume that changes in VIX have more impact on changes in SVIX than the other way around. By graphical inspection we can conclude that all the three series share the same major trends, which can be seen from Figure A.3 in Appendix. The full sample correlation between SVIX and VIX is 0.87 and for SVIX and VSTOXX it is 0.85. To more formally assess whether VIX or VSTOXX have any effect on SVIX, I run the following regressions

\[
\Delta \text{SVIX}_t = \alpha_0 + \beta_1 \Delta \text{VIX}_t + \beta_2 \Delta \text{VIX}_{t-1} + \epsilon_t
\]  

(5.10)

\[
\Delta \text{SVIX}_t = \alpha_0 + \gamma_1 \Delta \text{VSTOXX}_t + \gamma_2 \Delta \text{VSTOXX}_{t-1} + \epsilon_t
\]  

(5.11)

\[
\Delta \text{SVIX}_t = \alpha_0 + \beta_1 \Delta \text{VIX}_t + \beta_2 \Delta \text{VIX}_{t-1} + \gamma_1 \Delta \text{VSTOXX}_t + \gamma_2 \Delta \text{VSTOXX}_{t-1} + \epsilon_t
\]  

(5.12)

Results are reported in Table 5.8. Equation 5.10 tests whether there is a contemporaneous and lead effect in VIX on SVIX. Equation 5.11 tests the same thing for VSTOXX on SVIX and 5.12 includes both VIX and VSTOXX with first lags. ∆VIX has the highest contemporaneous effect on ∆SVIX with a coefficient equal to 0.317 and significant at a 1% level. The first lag of ∆VIX is also included to test whether there is a lead effect of VIX at time \( t - 1 \). Since there is a time-zone difference between the US and European markets, it might be good to clarify the contemporaneous and lead concepts. Since closing prices are used for all indices, the contemporaneous effect is the same calendar day even if Swedish and US markets are only coinciding for a few hours. Changes in VIX together with the first lag seems to explain some variation in the SVIX series, however the adjusted R² is very low, at 0.027, indicating that the independent variables do not explain much of the variation in SVIX. The results from the second regression are a bit

\[^5\text{I re-ran the same regression including up to three lags, however only the first lag shows any significance and the coefficients are small.}\]
surprising as one could expect that the implied volatility in the Swedish market should be affected by changes in European implied volatility indices, however, my results do not point at this. There seems to be neither any contemporaneous effect nor a lead effect from the VSTOXX on SVIX since both coefficients are insignificant at a 10% level together with an adjusted $R^2$ near zero. However, these results are in line with some previous research e.g. Ciffarelli and Paladino (2012) on volatility spillovers and Aboura (2003) on implied volatility index correlations. My results are also similar to the one derived by Skiadopoulos (2004) that there is a contemporaneous spillover of changes in GVIX and VIX, but no lead effect.

Table 5.8: Regression results - Volatility transmission

<table>
<thead>
<tr>
<th>Eq.</th>
<th>intercept</th>
<th>$\Delta VIX_t$</th>
<th>$\Delta VIX_{t-1}$</th>
<th>$\Delta VSTOXX_t$</th>
<th>$\Delta VSTOXX_{t-1}$</th>
<th>adj $R^2$</th>
<th>F-val</th>
<th>N</th>
</tr>
</thead>
<tbody>
<tr>
<td>(5.10)</td>
<td>0.039</td>
<td>0.317***</td>
<td>-0.083**</td>
<td>0.027</td>
<td>34.4</td>
<td>2,419</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.075)</td>
<td>(0.042)</td>
<td>(0.042)</td>
<td>(5.11)</td>
<td>0.012</td>
<td>0.055</td>
<td>-0.059</td>
<td>0.000</td>
</tr>
<tr>
<td></td>
<td>(0.073)</td>
<td>(0.041)</td>
<td>(0.041)</td>
<td>(5.12)</td>
<td>0.049</td>
<td>0.388***</td>
<td>0.005</td>
<td>-0.117*</td>
</tr>
<tr>
<td></td>
<td>(0.060)</td>
<td>(0.106)</td>
<td>(0.134)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Numbers in parentheses denote t-statistics based on Newey-West HAC robust standard errors. *, **, *** denotes significance at 10 percent, 5 percent and 1 percent respectively.

5.7 The relationship between SVIX and SSVIX

As pointed out in the Theory section, there are some limitations in the replication technique of variance swaps. Following Martin (2013) I construct the SSVIX index, which relies on simple variance swap replication, an instrument which can be priced and hedged even if we allow for jumps in the price of the underlying asset. According to Martin (2013), an index constructed in this way is a better measure of market volatility, compared to SVIX, which rather measures risk-neutral entropy of simple returns. In my sample, 91% of my SVIX observations are higher than SSVIX, which is a result in line with Martin (2013). After cleaning out my dataset I am left with 36,348 call options and 55,101 put options for the variance calculation, while Martin has 1,165,585. By having far less observations it might be reasonable to assume that some noise have been introduced into the SVIX and SSVIX series which might account for some days where SSVIX actually is higher than SVIX. Figure 5.4 shows the 10-day moving average of SVIX and SSVIX to the left and SVIX minus SSVIX to the right. From the right figure,

*This also relates to the discretization error since options cannot be traded at continuous strikes, so the integrals needs to be approximated by a sum. When there are non-tradable or non-existent strikes, the approximation error increases. More on this can be found in Martin (2013).
it is obvious that SVIX spikes were of larger magnitude than SSVIX during market turmoil periods. The MA differenced series reaches a max value of around 8, while in absolute terms for the ordinary series, differences are up to 20 ppts.

Figure 5.4: 10-day MA of SVIX and SSVIX and their differences

The fact that SVIX is higher in almost every case is evidence that we do not live in a log-normal world. According to Martin (2013), SVIX is more sensitive to the left tail of the return distribution, while SSVIX is more sensitive to the right tail. By comparing Equation (3.6), which points out that SVIX loads more on out-of-the-money puts, with (3.8), which shows that the SSVIX loads equally on options of all strikes, we get a hint of this. In the case when the left tail of the return distribution of the underlying asset is fatter than the right tail, SVIX will be higher than SSVIX. Therefore, as showed theoretically by Martin (2013), the difference between SVIX and SSVIX will be a measure of non-normality. He also points out that this evidence is much stronger than, what might be more familiar, the observation that histograms of log returns are not Normal.

Martin continues with connecting the SSVIX to the equity premium. He states that SSVIX can provide a lower bound on the equity premium and also is a measure of forward-looking expected excess return on the market under the true, as opposed to risk-neutral, probability distribution. What do these findings mean in terms of portfolio choices and trading strategies? If one finds the lower bound on the equity premium, according to the derivation in Martin (2013), to be very high compared to previous periods, one would receive extraordinary high premiums if selling options. Following the results derived by Martin, that SSVIX actually represents the real-world risk measure, we could develop a profitable trading strategy during times when SVIX-SSVIX difference spikes. We would then receive larger premiums than we should, under a real-world risk measure.
Chapter 6

Conclusion

6.1 Concluding remarks

In this paper I construct and implied volatility index, SVIX, from OMXS30 European-style options measuring the market expected 30 day forward volatility. My first objective was to assess whether this index could be constructed using the CBOE method for the Swedish market between January 3 2005 to June 11 2015. After having constructed the index, I wanted to outline its statistical properties, how it relates to the underlying index, how well it forecasts future volatility and whether it is affected by other international volatility indices. In addition, I construct a second index called SSVIX with a slightly different theoretical background, relying on simple variance swap replication.

SVIX is an index quite similar to other implied volatility indices. It is characterized by common properties of volatility series, e.g. long memory components, non-normality, mean-reversion and volatility clustering. I do not find any significant asymmetric effect in SVIX for the entire sample. However, there seems to be some tendency of such an effect in low-volatility regimes. The relationship between SVIX and OMXS30 is negative and of unidirectional causality, where changes in OMXS30 cause changes in SVIX. SVIX contains information about future volatility but provide a biased and inefficient forecast. From my sample, there seems to be evidence of implied volatility being superior to historical volatility, in forecasting future volatility.

There is a significant contemporaneous spillover effect on SVIX from VIX. In contrast to previous studies on the same subject (Dahlman and Wallmark (2007)) I find stronger effect from VIX than from the Eurozone implied volatility index, VSTOXX. When VIX_t and VIX_{t-1} are included as regressors, there seems to be both a contemporaneous and a lead effect at a 5% significance level, while no such effects are found for VSTOXX running
the separate regression. When both VIX and VSTOXX are included as regressors, only the contemporaneous VIX effect is significant at a 1% level.

Finally, the difference between SVIX and SSVIX is an index of non-lognormality, which exhibits spikes during market turmoil. The indices are computed based on two different theoretical approaches where SVIX measures entropy and SSVIX measures variance. The difference between SVIX and SSVIX is positive in 91% of the cases which could be taken as evidence that we do not live in a lognormal world. In theory, SSVIX can be higher than SVIX and would occur when returns are lognormally distributed under a risk-neutral measure.

\[1\] Following the derivation in Martin (2013)
Appendix A

Appendix

A.1 Normality tests

Figure A.1 shows histograms of SVIX and SSVIX and their differences series. As seen in the figure, the level series are right-skewed while the differenced series exhibit sincere kurtosis.

Figure A.1: Histograms of each respective serie
Appendix A.

A.2 Autocorrelation

By looking at $\Delta SVIX$ plotted in Figure A.2, we can expect the series to exhibit volatility clustering. Large changes tend to cluster together and small changes tend to cluster together, of either sign.

![Figure A.2: $\Delta SVIX$ plot](image)

A.3 Spillover effects in volatility indices - plot of VIX, VSTOXX and SVIX

Figure A.3 shows, as mentioned before, that VIX, VSTOXX and SVIX all share the same major trends. Correlations among indices ranges around 85%.

![Figure A.3: VIX in red, VSTOXX in blue and SVIX in dotted black](image)
A.4 Comparison of SVIX and SIXVX

The relationship between SVIX and SIXVX is very similar since both indices aim to measure the same thing, the 30 day expected volatility derived from option prices. What makes them two differ is the model and theoretical assumptions used for calculations. SIXVX is calculated through extracting the implied Black-Scholes volatility from ATM options. By extracting the implied volatility using Black-Scholes we rely on the same assumptions being made to derive the model. We know that some of those assumptions do not hold in practice. The one which is most significant is the constant volatility assumption, that the underlying asset volatility remains constant over strikes and over time. By assuming constant volatility, we find a "volatility smile/skew" if we plot the Black-Scholes implied volatility on the y-axis and moneyness on the x-axis. The Black-Scholes model also assumes that returns are lognormally distributed, which Martin (2013) shows is not the case and something we have seen many times before. These two assumptions are the ones which are often shown to not hold in the real world. For a full derivation of the assumptions of the Black-Scholes model, I refer to Hull (2000).

Figure A.4 shows an example of the implied volatility surface, with moneyness and time-to-maturity on the axis.

\[
\text{Implied Volatility Surface}
\]

\[
\sigma(T, M)
\]

As seen from Figure A.5 the movements between SVIX and SIXVX are very similar. The sample correlation is 0.92. The bias in SIXVX comes from the fact that it is based on Black-Scholes model. More on this bias can be found in Carr and Wu (2006) which makes the same analysis on VIX and VXO.
A.5 SVIX - OMXS30 relationship

Table A.1 shows correlation coefficients for different sub periods. Previous studies have conducted that correlations between volatility and index returns are higher in high volatility regimes. My results slightly indicates the same conclusion since sub periods 1 and 6 are low volatility regimes and have slightly lower correlation, while medium and high sub periods (2, 3, 4 5) show larger correlation between ∆SVIX and OMXS30 returns.

**Table A.1:** sub period correlations between ∆SVIX and OMXS30 CC daily returns

<table>
<thead>
<tr>
<th>sub period</th>
<th>( \rho )</th>
<th>pValue</th>
</tr>
</thead>
<tbody>
<tr>
<td>#1</td>
<td>-0.30</td>
<td>0.000</td>
</tr>
<tr>
<td>#2</td>
<td>-0.31</td>
<td>0.000</td>
</tr>
<tr>
<td>#3</td>
<td>-0.33</td>
<td>0.000</td>
</tr>
<tr>
<td>#4</td>
<td>-0.38</td>
<td>0.000</td>
</tr>
<tr>
<td>#5</td>
<td>-0.34</td>
<td>0.000</td>
</tr>
<tr>
<td>#6</td>
<td>-0.27</td>
<td>0.000</td>
</tr>
</tbody>
</table>

Table A.2 shows the re-estimated asymmetry regression for each specific sub period. As seen, the coefficient for OMXret\(_t\) is negative in all cases, and significant at a 5% level in all periods except for the first, where the it is only significant at a 10% level. Sub period 6 is the only one showing a significant and positive asymmetry coefficient, and it is a low volatility period.
### Table A.2: Regression results - asymmetry

<table>
<thead>
<tr>
<th>subp.</th>
<th>intercept</th>
<th>OMXret</th>
<th>adj R²</th>
<th>N</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>-0.056</td>
<td>-0.386*</td>
<td>0.096</td>
<td>340</td>
</tr>
<tr>
<td></td>
<td>(0.121)</td>
<td>(0.197)</td>
<td>(0.314)</td>
<td></td>
</tr>
<tr>
<td>2.</td>
<td>-0.139</td>
<td>-0.572***</td>
<td>0.096</td>
<td>552</td>
</tr>
<tr>
<td></td>
<td>(0.205)</td>
<td>(0.173)</td>
<td>(0.278)</td>
<td></td>
</tr>
<tr>
<td>3.</td>
<td>-0.312</td>
<td>-0.871**</td>
<td>0.098</td>
<td>204</td>
</tr>
<tr>
<td></td>
<td>(0.869)</td>
<td>(0.362)</td>
<td>(0.599)</td>
<td></td>
</tr>
<tr>
<td>4.</td>
<td>-0.222</td>
<td>-0.708***</td>
<td>0.145</td>
<td>502</td>
</tr>
<tr>
<td></td>
<td>(0.198)</td>
<td>(0.189)</td>
<td>(0.333)</td>
<td></td>
</tr>
<tr>
<td>5.</td>
<td>-0.228</td>
<td>-0.462**</td>
<td>0.107</td>
<td>241</td>
</tr>
<tr>
<td></td>
<td>(0.331)</td>
<td>(0.224)</td>
<td>(0.351)</td>
<td></td>
</tr>
<tr>
<td>6.</td>
<td>0.357</td>
<td>-1.139***</td>
<td>0.086</td>
<td>666</td>
</tr>
<tr>
<td></td>
<td>(0.133)</td>
<td>(0.168)</td>
<td>(0.277)</td>
<td></td>
</tr>
</tbody>
</table>

Numbers in parentheses denote t-statistics based on Newey-West HAC robust standard errors. *, **, *** denotes significance at 10 percent, 5 percent and 1 percent respectively.

### A.6 Option data used in the SVIX/SSVIX calculation

Table A.3 show the number of call an put options used to calculate my SVIX and SSVIX series. As discussed in Chapter 4, I dropped those days where only one $\Delta K$ could be calculated. For the call options, this was 20 days and for the puts seven days. None of the days coincide, i.e. there is no day where this restriction makes it impossible to calculate SVIX or SSVIX. Table A.3 shows the actual number of options used to calculate my 2,505 SVIX/SSVIX values.

<table>
<thead>
<tr>
<th></th>
<th>Calls</th>
<th>Puts</th>
</tr>
</thead>
<tbody>
<tr>
<td>OOTM</td>
<td>36,348</td>
<td>55,101</td>
</tr>
<tr>
<td>ATM/ITM</td>
<td>26,303</td>
<td>23,800</td>
</tr>
<tr>
<td>Total</td>
<td>62,651</td>
<td>78,901</td>
</tr>
</tbody>
</table>


